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APPLICATION OF MULTIDIMENSIONAL UTILITY THEORY
IN DETERMINING OPTIMAL TEST-TREATMENT STRATEGIES
FOR STREPTOCOCCAL SORE THROAT AND RHEUMATIC FEVER

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In most decision problems there is a need to consider both multiple objective function and uncertainty. Von Neumann-Morganstern utility theory offers a rigorous methodology for defining objectives under uncertainty, and there is nothing in the theory which precludes construction and use of multiple criteria utility functions. For some types of problems feasible methods now exist for overcoming the severe practical problems of assessing such functions. This paper describes the analysis of a medical problem, the

determination of optimal test and treatment strategies for streptococcal sore throat and rheumatic fever, in which the formulation, verification, and assessment of a multidimensional utility function was a critical part of the analysis. Comments on the usefulness of the procedure are made, and some potential future applications summarized. A theorem on the assessability of parameters in the utility function is also proven.

APPLICATION OF MULTIDIMENSIONAL UTILITY THEORY IN DETERMINING OPTIMAL TEST-TREATMENT STRATEGIES FOR STREPTOCOCCAL SORE THROAT AND RHEUMATIC FEVER

William C. Giauque, D.B.A.
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I. Introduction

Analysis of medical problems is, in general, complicated by both the need to consider multiple objective criteria and the need to allow for uncertainty in diagnostic and treatment procedures. A general methodology for defining objective functions valid under uncertainty exists in the form of von Neumann - Morgenstern utility theory. Recent theoretical developments relating to the formulation and assessment of multidimensional utility functions offer a practical technique useful for multiple criteria. Some uses of these techniques have appeared in the applications literature, including some descriptions of medical applications. Generally, however, these applications utilize assumptions leading to the additive form of the utility function, thus avoiding a number of assessment problems inherent in more complicated forms. In this paper, we describe the analysis of a medical problem of considerable interest, the prevention and treatment of rheumatic fever. In this work, the assumptions leading to the additive form of the utility function were not justified, but it was possible to utilize a less restrictive form, the multiplicative form. Techniques for assessing the parameters of this form are described, and some general comments concerning the usefulness of such techniques in medical problems are made. A more complete discussion of this work can be found in Giauque[10].

II. The Medical Problem

Rheumatic fever and associated rheumatic heart disease remain significant health problems in the United States today, despite the existence of effective medical means of preventing the disease. In 1968 about 16,000 deaths from this disease were reported in the United States, a rate far exceeding annual death rates from poliomyelitis (2893 in 1952) and measles (364 in 1963) in the years preceeding the massive immunization efforts against these diseases[20]. In 1970, it was estimated that about 100,000 new cases of rheumatic fever and rheumatic heart disease occur each year[29].

Rheumatic fever is closely associated with streptococcal infections, generally of the upper respiratory tract. "Strep throat", as most parents of school age children know, is one of the most common diseases of childhood, accounting for more child-hours lost from school than any other acute bacterial illness[20]. Streptococcal infections themselves can be acute, resulting in considerable discomfort and, if untreated, requiring about five days of bed rest, or can be so mild that the patient doesn't realize he is infected. Treatment usually gives almost immediate relief from disease symptoms[20]. In most cases a child with a strep infection recovers completely, but in a small percentage of cases rheumatic fever, or even more rarely a kidney disease known as acute glomerulonephritis, develops. Both of these diseases can result in long term disability and/or death.

Streptococcal infections are relatively easy to treat, as streptococci are extremely sensitive to penicillin and related antibiotics. A ten day course of oral penicillin or a single injection of the drug is generally sufficient to eradicate the infection. Rheumatic fever, on the other hand, is poorly understood and difficult or impossible to treat. Generally a rheumatic patient is given an antibiotic

(usually penicillin) to eradicate any strep infections which may be present and put to bed for a period of time lasting up to three months (more in some cases). There is some evidence that treatment with salicylates or steroids may shorten or alleviate the course of the disease, but the effectiveness of such treatments is controversial, and at best is limited[17]. Acute glomerulonephritis is likewise an extremely difficult disease to treat.

A number of studies have shown that successful eradication or prevention of strep infections significantly lower occurrence rates of both rheumatic fever and acute glomerulonephritis[1,21]. Thus, it appears that a major barrier to lowering the incidence of these diseases is the lack of a method of determining when antistreptococcal treatment is warranted. This raises a number of the questions which were addressed in this study. First, a number of strep infections never become acute, thus are not brought to the attention of the medical community. Is it worth conducting community-wide surveys of school children to detect such cases? Is it worth checking all members of a strep patient's family to isolate additional strep infections? Secondly, when a child develops an acute sore throat, it is by no means certain that it is caused by streptococci, as a number of viral infections can cause similar symptoms. Penicillin is ineffective against viral infections. The primary clinical tool available in differentiating these agents is the throat culture, but results of this test are not available for at least twenty-four hours. Thus, when confronted with an acute sore throat, a clinician has at least two decisions to make: (1) should he take a throat culture or not; and (2) should antibiotic treatment of the illness be delayed until the results of the throat culture become known? In this latter decision, one must weigh the relative risk of starting

antibiotic treatment early and perhaps uselessly versus the risk of delaying treatment at least 24 hours to be sure of the diagnosis, thus adding to the danger of contracting rheumatic fever or nephritis. Finally, there has been some controversy in the medical literature over the propriety of maintaining former rheumatic patients on a continuous program of penicillin medication for life, as is normally recommended.

III. Analytical Approach

A. Specification of the Result Vector

In investigating the questions outlined above, it became clear that any final measure of outcome would have to include a number of factors affecting the patient. For example, the decision to begin antibiotic treatment affects not only the risk of rheumatic fever and the danger of an antibiotic reaction, but the length and severity of the sore throat and the cost of the treatment. A list of all factors considered important in this problem is given in Table I. The dimensionality of this result vector was first reduced as much as possible by simple trade-off arguments. For example, a measure of total dollar cost to the patient was derived by summing all the patient's direct and indirect costs. It was assumed that the exact allocation of these costs among the various uses was much less important than the total of all the costs. After these simplifications, the result vector contained ten dimensions summarized in table II. It was now necessary to define a utility function over this multidimensional result space.

TABLE I. COMPLETE VECTOR OF RESULTS

I. Factors Related to Dollar Costs
  A. Direct cost of the treatment to the practice
  B. Amount of doctor's time used
    1. Direct cost
    2. Opportunity cost
  C. Amount of nurses' time used
    1. Direct cost
    2. Opportunity cost
  D. Estimated overhead expense of the practice allocated to the case
  E. Amount billed to the patient
    1. Amount paid out-of-pocket by the patient
    2. Amount paid by insurance or welfare plans
  F. Costs not billed to the patient by government, insurance, or welfare plans
  G. Amount of parent's and patient's time used
    1. Direct cost (lost wages or profits, babysitting fees, etc.)
    2. Opportunity cost
  H. Patient's other expenses as a result of treatment
    1. Medical (prescripticns, supplies, etc.)
    2. Nonmedical (transportation, etc.)

II. Factors Related to Health
  A. Immunity developed to one strain of strep (yes or no)
  B. Days ill with strep throat (zero to five)
    1. Child
    2. Sibling
    3. Other
  C. Method of receiving medication
    1. Single injection
    2. Pills three times per day for ten days
  D. Antibiotic reactions
    1. Immediate death
    2. Severe
    3. Moderate
    4. Mild
    5. None
    6 - 25. All combinations of initial occurrences described by 2-5 plus second occurrences described by 1-5.
  E. Acute rheumatic fever episode
    1. Severe
    2. Mild
  F. Prophylactic regimen for rheumatic fever patients
    1. Episodal - treat each strep recurrence
      a. Single injections
      b. Pills three times per day for ten days
  G. Long term rheumatic fever effects
    1. Death by age 21
    2. Severe damage
    3. Moderate damage
    4. Mild damage
    5. No damage
  H. Severity of glomerulonephritis
    1. Death
    2. Chronic nephritis
    3. Complete recovery
    4. None

TABLE II. CONSOLIDATED VECTOR OF RESULTS

Dimension	Description
x_1	Cost to the patient
x_2	Cost (or profit) to the doctor
x_3	Cost to the public or insurance system
x_4	Method of medication if any (oral or injected)
x_5	Immunity developed to infecting strain of strep - yes or no)
x_6	Days ill with strep infection
x_7	Antibiotic reaction
x_8	Severity of acute rheumatic fever episcde
x_9	Type of post rheumatic medication, if any
x_{10}	Long term effects of rheumatic fever

B. Specification of the Utility Function

For any but the most trivial cases, direct assessment of a general multidimensional utility function is out of the question. Humans are poor at making trade-offs in multidimensional spaces and tend to rely on lexicographic procedures, thus making a direct assessment suspect. In addition, the sheer number of judgments which would have to be elicited to define a general utility function over more than two, or perhaps three, dimensions would make this approach impractical. These problems could be simplified if one could represent a multidimensional utility function as a function of many unidimensional functions. Symbolically, one would like to write

$$u(\underline{x}) = f[u_1(x_1), u_2(x_2), \ldots, u_n(x_n)] \qquad (1)$$

where $\underline{x} = (x_1, x_2, \ldots, x_n)$ is a particular consequence from consequence space $\underline{X} = X_1 \cdot X_2 \cdot \ldots \cdot X_n$, $u(\underline{x})$ is the utility of $\underline{x}$, and the $u_i(x_i)$ are the utilities of each of the x_i. Each unidimensional function $u_i(x_i)$ could be assessed separately by standard techniques, and assuming that the functional form of (1) were known, complete assessment of $u(\underline{x})$ would be possible.

There are a number of structural assumptions which lead to such a representation. Three assumptions investigated in the course of this research are utility independence, pairwise preferential independence, and pairwise marginality, as defined by Keeney[12,13,14,15]. These terms are defined and the resulting functional forms summarized

here for convenience. A more extensive discussion of these concepts can be found in the references discussed in the Note at the end of this paper.

III.B.1. Utility Independence

One speaks of one particular dimension of a consequence space, say dimension x_i, as being utility independent of the remaining dimensions if the decision maker's utility curve over x_i is the same (within a positive linear transformation) for all values of the remaining dimensions. Formally stated, define $\underline{X}_{i-} \equiv X_1 \cdot X_2 \cdot \ldots \cdot X_{i-1} \cdot X_{i+1} \cdot \ldots \cdot X_n$, and let $\underline{x}_{i-}$ be a member of $\underline{X}_{i-}$. Then X_i is utility independent of $\underline{X}_{i-}$ if one's preference order over lotteries on X_i with $\underline{X}_{i-}$ held fixed does not depend on the fixed amount $\underline{x}_{i-}$. If X_i is utility independent of $\underline{X}_{i-}$ for $i = 1, 2, \ldots, n$ then order one mutual utility independence is said to hold. In this case, Keeney[15] shows that (1) takes a quasi-additive form

$$u(\underline{x}) = \sum_{i=1}^{n} k_i u_i(x_i) + \sum_{i=1}^{n} \sum_{j=i+1}^{n} k_{ij} u_i(x_i) u_j(x_j) + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} k_{ijk} u_i(x_i) u_j(x_j) u_k(x_k) + \ldots \tag{2}$$

where the k_i, k_{ij}, k_{ijk}, ... are constants between zero and one.

Although this result allows a representation such as (1), the number of constants which must be assessed in (2) is excessively large for many practical problems. If n = 6, for example, one would have to evaluate 41 constants. More useful is the result summarized below.

III.B.2. Pairwise Preferential Independence

Pairwise preferential independence is said to hold if the trade-offs one is willing to make between attributes taken two at a time are not dependent on the values of the remaining attributes. Formally stated, define $\underline{X}_{ij-} \equiv X_1 \cdot \ldots \cdot X_{i-1} \cdot X_{i+1} \cdot \ldots \cdot X_{j-1} \cdot X_{j+1} \cdot \ldots \cdot X_n$, and let x_{ij-} be a particular value from $\underline{X}_{ij-}$. Then $X_i \cdot X_j$ is pairwise preferentially independent of $\underline{X}_{ij-}$ if one's preference order for consequences (x_i, x_j, x_{ij-}) with x_{ij-} held fixed does not depend on the fixed amount x_{ij-}. Keeney[14] then shows that $u(\underline{x})$ can be represented by one of the following forms:

$$u(\underline{x}) = \sum_{i=1}^{n} k_i u_i(x_i) \text{ (additive form), or} \tag{3}$$

$$1 + ku(\underline{x}) = \prod_{i=1}^{n} [1 + kk_i u_i(x_i)] \text{ (multiplicative form)}, \tag{4}$$

where k and the k_i are constants with $0<k_i<1$ and $k>-1$. In both these cases the assessment requirements are reasonable even for consequence spaces of fairly high dimensionality. The additive and multiplicative cases can be distinguished on the basis of the following property.

III.B.3. Pairwise Marginality

If the decision maker's preferences for gambles depends only on the marginal distributions of the consequences, rather than the joint distributions, then the additive form of the utility function (4) holds[3,5,6,7,8,22]. A convenient test for this property is the pairwise marginality test. Let x_i^1, x_i^2, x_j^1, and x_j^2 be distinct values of x_i and x_j, and let x_{ij-} take on some constant value. Define gambles A and B as follow:

gamble A yields (x_i^1, x_j^1, x_{ij-}) with prob .5

and (x_i^2, x_j^2, x_{ij-}) with prob. .5; and

gamble B yields (x_i^1, x_j^2, x_{ij-}) with prob. .5,

and (x_i^2, x_j^1, x_{ij-}) with prob. .5 .

If the decision maker is indifferent to the gambles, then pairwise marginality holds between attributes x_i and x_j. If pairwise marginality holds between all i and j, then (3) holds.

III.C. Utility structure Verification

To illustrate the verification of utility independence, consider the dimension "days ill with strep infection". The maximum and minimum number of days possible were determined to be ten and zero, respectively. Utility independence was verified by the following kinds of questions:

> "Suppose values of all other dimensions are specified (ie. the cost to the patient, doctor, and insurance systems, immunity developed, method of medication, severity of antibiotic reaction, etc. are

all given). Now consider the gamble

no days ill with prob. p,

ten days ill with prob. 1-p

and determine a number x such that if you had to choose either the gamble or the x days ill for sure, you would be indifferent. Now suppose a different set of values for the other dimensions is given and ycu are presented with the same gamble and asked to assess x again. Does the value of x change?"

If the value of x doesn't change no matter what values are given for the other dimensions, and if this is true for all gambles on the "days ill" dimension, then the "days ill" attribute is utility independent of the other attributes. Utility independence of the remaining dimensions can be similarily verified.

To illustrate the verification of preferential independence, consider the dimensions "cost to the patient" and "days ill with strep infection". In verifying preferential independence, we are attempting to determine whether or not we need to consider values of the remaining attributes when making trade-offs between cost to the patient and days ill. The following dialogue illustrates this process.

"Consider a consequence, which we'll call consequence A for convenience, involving

($100 cost to the patient, 5 days ill with strep infection, and some previously specified values for all the other attributes).

Now determine a dollar figure x such that consequence B, defined as

($x cost to the patient, no days ill with strep infection, and same values for other attributes as consequence A)

is exactly as attractive as consequence A. Now change the values of some or all of the attributes other than

cost and days ill. Does the value of x change?"
If the value of x doesn't change for all values of the other attributes, and if this holds true for all trade-offs between dollars and days ill, then these two attributes are preferentially independent of the other attributes. Preferential independence between other pairs of attributes is determined in a similar manner.

In the strep - rheumatic fever problem, both utility independence and preferential independence were verified over the entire attribute space. The property of pairwise marginality was not, however, found to hold between all pairs of attributes. For example, the gambles A and B below, involving combinations of "antibiotic reaction" and "long term effects of rheumatic fever" were not, in general, equally preferred.

Gamble A: (no reaction, no long term effects) with prob. .5 ; and
(severe reaction, severe rheumatic damage) with prob. .5 .

Gamble B: (no reaction, severe rheumatic damage) with prob .5 ;
and (severe reaction, no long term effects) with prob. .5 .

Pairwise marginality was found to hold only if the two attributes involved were attributes x_1 (cost to the patient), x_2 (cost or profit to the doctor), x_3 (cost to the public or insurance system), and x_4 (method of medication). These results, together with the utility independence and preferential independence properties already verified, imply the following form of the utility function:

$$u(\underline{x}) = k_1 u_1(x_1) + k_2 u_2(x_2) + k_3 u_3(x_3) + k_4 u_4(x_4) + k_r u_r(\underline{x}_r) , \qquad (5)$$

where

$$\underline{x}_r = (x_5, x_6, x_7, x_8, x_9, x_{10}) ,$$

and

$$1 + ku(\underline{x}_r) = [1+kk_5u_5(x_5)] \ldots [1+kk_{10}u_{10}(x_{10})] .$$

Again, k and the k_i are constants with $k > -1$ and $0<k_i<1$, $i = 1, \ldots , 10$.

IV. Assessment of the Parameters of the Utility Function

In general if the additive form of the utility function holds for a result vector of dimensionality n, the constants in (3) can be evaluated by choosing n linearly independent values $\underline{x}^1, \ldots , \underline{x}^n$ of the result vector and directly assessing the utilities $u(\underline{x}^i)$ for $i = 1, \ldots , n$. One thus obtaines n linearly independent equations in the unknown k_i, and the values of the constants are readily obtained. The values of $\underline{x}^i$ used in this determination must be carefully chosen, however, as it is difficult to assess a consistent utility measure over multidimensional consequences. A useful scheme is as follows. Let x^0_j represent some natural "base" level of dimension x_j , and let x^c_i represent some value of x_i different from x^0_i . Define $\underline{x}^i = (x^0_1, \ldots , x^0_{i-1}, x^c_i, x^0_{i+1}, \ldots , x^0_n)$ and assess $u(\underline{x}^i)$. This enables the decision maker to concentrate on one dimension at a time in assessing the $u(\underline{x}^i)$.

Particularly useful schemes involve setting $x_j^o = x_{j*}$ (or x_j^*) and $x_i^c = x_i^*$ (or x_{i*}), where x_j^* and x_{j*} represent the most and least desirable outcomes of consequence j . The unidimensional utility functions $u_j(x_j)$ are generally scaled so that $u_j(x_j^*) = 1$ and $u_j(x_{j*}) = 0$.

In the multiplicative case the same ideas apply, except that n+1 parameters must be estimated. If one can set x_j^o equal to x_{j*} and x_i^c equal to x_i^* , as discussed above, values of the constants are easily obtained since a series of n equations of the form

$$1 + ku(\underline{x}^i) = 1 + kk_i$$

result, so the k_i are determined immediately. The value of k can then be obtained from the consistency equation

$$1 + k = \prod_{i=1}^{n} (1 + kk_i) ,$$

which results when $\underline{x} = (x_1^*, \ldots , x_n^*)$.

In some situations the above procedure may be difficult to apply. In medical problems, for example, many of the x_i could represent dimensions describing health, so the x_{i*}'s would represent various extremes of ill health. In order to assess $u(x_{1*}, \ldots , x_i^c, \ldots , x_{n*})$ one would have to

consider his preferences under the assumption that his health was at the worst possible state in all attributes but one, a procedure which involves obvious difficulties. It would be more natural in this case to assess utilities of the form $u(x_1^*, x_2^*, \ldots, x_i^c, \ldots, x_n^*)$, corresponding to the case where the patient is in perfect health along all dimensions but one. In order to recover the scaling parameters under this scheme, one must solve a series of n+1 nonlinear equations in n+1 unknowns. It can be shown that there is exactly one solution to these equations which yields scaling parameters in the feasible range. Furthur, a simple search procedure which locates that solution can be defined. A proof of these statements can be found in the Appendix.

IV. Results

The unidimensional utility functions and scaling parameters necessary to specify the utility function for this problem were determined, through interviews, for two doctors, three nurse - practitioners (nurses specially trained to handle a variety of routine diagnostic and therapeutic situations), three public health officials, and five patients. We were interested not only in answering the medical questions raised in this research but in determining the stability of the medical recommendations resulting from the utility assessments, and in investigating systematic differences in the utilities of members of the different assessing groups, particularly if those differences affected recommended treatments. Optimal decisions were obtained for each of the respondants by using each assessed utility function as the objective criterion in a dynamic programming

algorithm.

It was found that there was almost total agreement among respondents on the proper course of medical care implied by the utility functions, although the assessed utility functions differed considerably. We also found that within the detection limits dictated by our small sample size, there were no systematic differences in the utilities assessed by the different groups. Differences of individual utilities within groups were much larger than differences among groups.

The model strongly indicated that oral penicillin therapy should be started immediately if there were the slightest suspicion of strep infection. Throat cultures should be taken on all patients, even if the perceived risk of strep infection was small, and if the culture results proved negative, therapy should be discontinued on those patients on antibiotics. It was also shown that routine community-wide streptococcal screenings should be performed. Once a patient has rheumatic fever, current medical practice indicates that he should be kept under continual penicillin (or other antibiotic) medication, and the model results confirmed the propriety of this practice, at least until the patient reaches twenty-one years of age. The age of the patient when the initial strep infection is suspected and the length of time he waits to see the doctor after becoming ill have no effect on any of these recommendations. Finally, the relative effectiveness of a physician was compared to that of a nurse - practitioner in the diagnosis and care of strep infections, with the result that the nurse was shown to be at least as effective as the physician.

V. Conclusion and Implications

Decision problems in medicine have become increasingly complex and important in recent years, both because of rapidly expanding knowledge and technology, and higher stakes in terms of money and human life. The modern medical doctor has more tools at his disposal in combating illness than at any time in history, but often the knowledge of how the tools can best be used is lacking. The sheer volume of new medical knowledge makes it difficult to integrate the knowledge into a consistent system for treating illness which is in some sense optimal, and communicating the system to practicing medical personnel. In addition, the traditional organization of medical knowledge along the lines of specific diseases, their symptoms, and their treatments makes it difficult to integrate new techniques which may affect differential diagnosis or recommended treatments for a number of diseases.

The analysis outlined in this paper demonstrates the usefulness of the multidimensional utility approach to defining objective functions in situations of this complexity. In combination with the other tools of decision analysis, this forms a powerful, flexible method of analysis allowing systematic inclusion of new knowledge in the canon of accepted medical practice. The complexity and time demands of the analysis probably preclude patient-by-patient analysis, but the methods offer great promise in determining recommended practices for dealing with entire groups of patients with specific groups of symptoms. There is some evidence that protocols can be developed for many disease systems which are structured enough for unskilled personnel to apply in routine medical examinations. Another use of this analytical technique is suggest by Forst[9], who offers the interesting idea of using a decision analytic approach to determine malpractice settlements. A multidimensional utility objective would, in general, be necessary in such an approach. Finally, the means offered of trading off many

conflicting factors and evaluating the utility of combinations of factors may offer a workable scheme for defining a quality of care measure.

Note:

References 4, 11, 16, 24, 25, and 27 discuss the basic theory of utility functions, with particular emphasis on application methodology for unidimensional utility functions. The concept of risk aversion, particularly important in utility functions over monitary consequences, is discussed and developed in references 2 and 23. Assessment procedures to insure certain desirable properties are discussed in references 19, 24, 25, and 28. Reference 26 discusses assessment procedures and application methodologies. A number of the results cited in the paper also appear in various forms in references 3, 5, 6, 7, 8, 18, and 22.

APPENDIX - DETERMINATION OF THE ROOTS OF THE PARAMETER EQUATIONS

Consider the general assessment scheme where n+1 vectors $\underline{x}^m = (x_1^m, \ldots, x_n^m)$ are chosen and $u(\underline{x}^m)$ assessed for each $m=1,2,\ldots,n+1$. The n+1 equations

$$1 + ku(\underline{x}^m) = \prod_{j=1}^{n} [\ 1 + kk_j u_j (x_j^m)\ ]$$

contain the information necessary to solve for the parameters k and k_i, $i=1,\ldots,n$ if the $\underline{x}^m$ are independent. The equations can be simplified if the $\underline{x}^m$ are chosen such that

$$\underline{x}^m = \underline{x}^i = (x_1^0, \ldots, x_{i-1}^0, x_i^c, x_{i+1}^0, \ldots, x_n^0)$$

for $m=1,\ldots n$ and

$$\underline{x}^{n+1} = \underline{x}^0 = (x_1^0, \ldots, x_n^0)$$

where the x_j^0 represent arbitrary but "natural" base levels and $x_i^c \neq x_i^0$. One then obtains one equation of the form

$$1+ku(\underline{x}^0) = \prod_{j=1}^{n} [\ 1 + kk_j u_j (x_j^0)\ ] \tag{6}$$

and n equations of the form

$$1+ku(\underline{x}^i) = A_i \prod_{j=1}^{n} [\ 1 + kk_j u_j (x_j^0)\ ]\ ,\ i=1,\ldots n \tag{7}$$

where

$$A_i = [\,1+kk_i u_i (x_i^c)\,] \,/\, [\,1+kk_i u_i (x_i^0)\,]\ .$$

Substituting equation (6) for the multiplicative term in (7) gives

$$1+ku(\underline{x}^{i}) = A_{i} \cdot (1+ku(\underline{x}^{o})) \tag{8}$$

which can be solved for k_i, giving

$$k_{i} = \frac{u(\underline{x}^{i}) - u(\underline{x}^{o})}{u_{i}(x_{i}^{c}) - u_{i}(x_{i}^{o}) + k[u_{i}(x_{i}^{c})u(\underline{x}^{o}) - u_{i}(x_{i}^{o})u(\underline{x}^{i})]} . \tag{9}$$

Substituting this result into (6) gives the n^{th} order equation in k

$$1 + ku(\underline{x}^{o}) = \prod_{i=1}^{n} \frac{[u_{i}(x_{i}^{c}) - u_{i}(x_{i}^{o})] \cdot [1 + ku(\underline{x}^{o})]}{[u_{i}(x_{i}^{c}) - u_{i}(x_{i}^{o})] + k[u_{i}(x_{i}^{c})u(\underline{x}^{o}) - u_{i}(x_{i}^{o})u(\underline{x}^{i})]} . \tag{10}$$

If we let $x_{i}^{o} = x_{i}^{*}$ and $x_{i}^{c} = x_{i*}$ and scale the utility functions so that $u_{i}(x_{i}^{*}) = 1$ and $u_{i}(x_{i*}) = 0$, then (10) simplifies to

$$1 + k = \frac{(1+k)^{n}}{\prod_{i=1}^{n} [1 + ku(\underline{x}^{i})]} . \tag{11}$$

It can be shown that there is exactly one value of k greater than -1 such that equality holds. This result is put in the form of the following theorem:

Theorem:*

* The motivation and outline of this proof are due to

Professor Richard F. Meyer of the Harvard Business School.

There is exactly one root k of (11) greater than -1. Furthur, the root lies in $(-1,0)$ or $(0,\inf)$ depending on whether $\sum_{i=1}^{n}[1 - u(\underline{x}^{i})]$ is less than or greater than unity, respectively.

Proof:

Let $z = 1+k$ and $v_i = u(\underline{x}^{i})$. Equation (11) becomes

$$z = z^{n} / \prod_{i=1}^{n} [1 - v_i + zv_i] . \qquad (12)$$

Now introduce $a_i = v_i(1-v_i)$ and substitute into (12), getting

$$\prod_{i=1}^{n} (1+za_i) = z^{(n-1)} \prod_{i=1}^{n} (1+a_i) \qquad (13)$$

and define the function

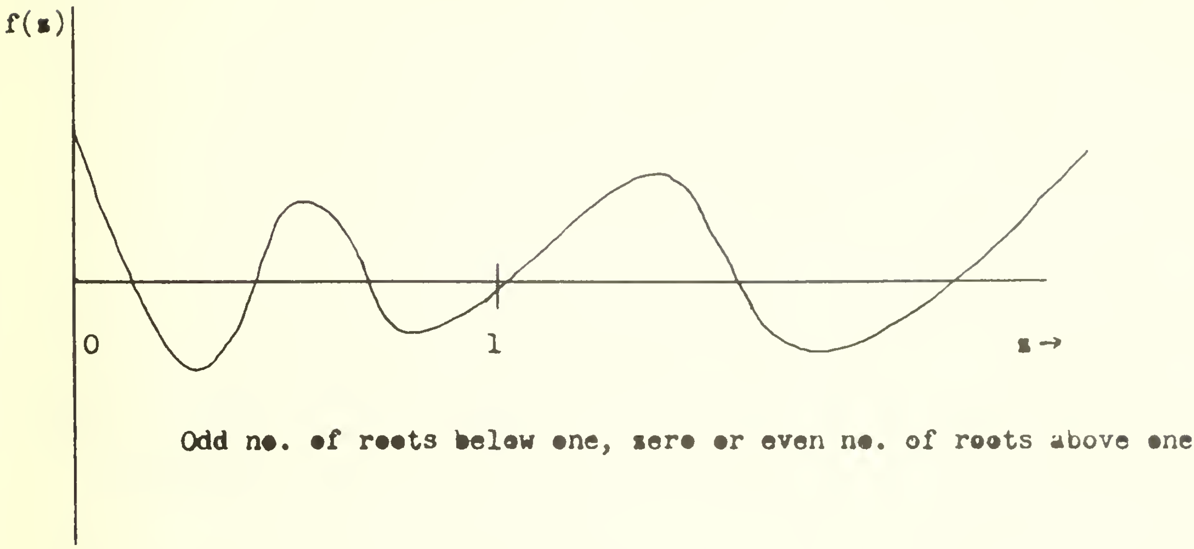
$$f(z) = \prod_{i=1}^{n} (1+za_i) - z^{(n-1)} \prod_{i=1}^{n} (1+a_i) . \qquad (14)$$

We now wish to know how many solutions $z>0$ exist such that $f(z)$ equals zero.

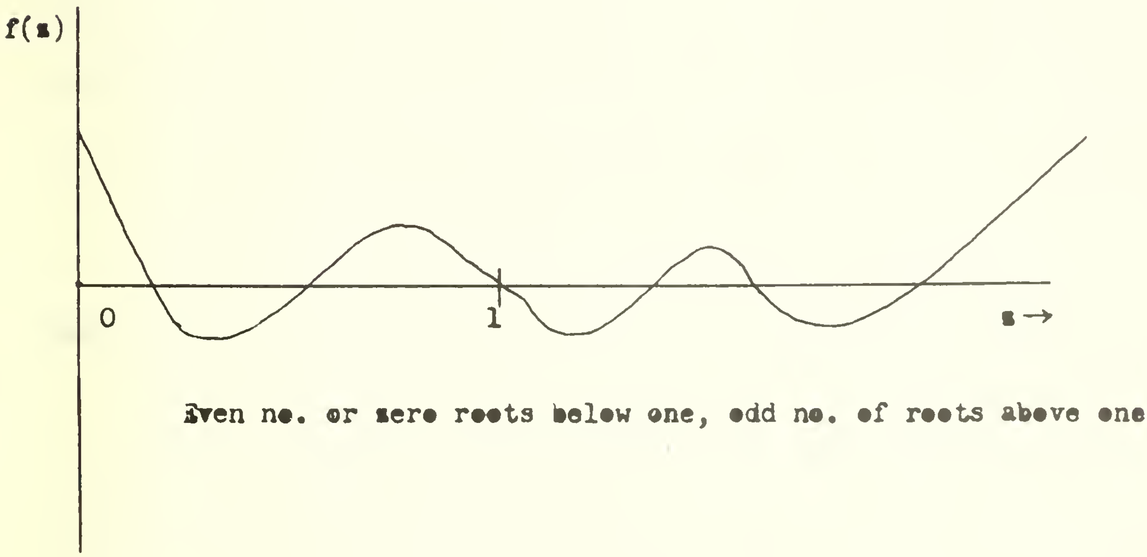
First note that one solution occurs at $z=1$. This is a degenerate case corresponding to $k=0$, in which case the utility function is additive. In the following, it will be assumed that $z\neq 1$. Also note that $f(0)$ and $f(\inf)$ are both positive since the a_i are positive if the v_i are restricted to $(0,1)$. Thus one of the two cases illustrated in Figure 1 must exist, depending on the slope of $f(z)$ at $z=1$. This slope is given by

Figure 1 - Shape of f(z)

Case a: f'(1) greater than zero



Case b: f'(1) less than zero



$$f'(z) = \prod_{i=1}^{n} (1+za_j) \sum_{i=1}^{n} [a_i/(1+za_i)]$$

$$-(n-1)z^{(n-2)} \prod_{i=1}^{n} (1+a_i) , \tag{15}$$

which gives, when evaluated at $z=1$,

$$f'(1) = \prod_{j=1}^{n} (1+a_j)[\sum_{i=1}^{n} a_i/(1+a_i)]$$

$$- (n-1) \prod_{i=1}^{n} (1+a_i)$$

$$= \prod_{j=1}^{n} (1+a_j)[\sum_{i=1}^{n} v_i - (n-1)] . \tag{16}$$

The sign of $f'(1)$ depends only on the sign of the term in the brackets. A physical interpretation for this quantity is discussed after the proof of the theorem is concluded.

If equation (14) is expanded and like terms in a_i collected, we derive

$$f(z) = A_0(z^n - z^{n-1}) + A_1(z^{n-2} - z^{n-1}) + \ldots$$

$$+ A_{n-2} (z-z^{n-1}) + (1-z^{n-1}) , \tag{17}$$

where A_0 through A_{n-2} are positive. Letting $y = 1/z$, substituting in (17) and multiplying by y^n gives

$$f(y) = (y^n-y) + A_{n-2} (y^{n-1} -y) + \ldots$$

$$+ A_1(y^2-y) + A_0(1-y) . \tag{18}$$

The second derivitive of $f(y)$ is given by

$$f''(y) = n(n-1)y^{n-2} + (n-1)(n-2)y^{n-3} A_{n-2} +$$

$$\ldots + 2A_1 \tag{19}$$

which is positive for all $y>0$. Thus $f(y)$ is convex for $y>0$ and can be zero at most twice. One zero is at $y=1$,

as mentioned above, and the other zero must lie in (0,1) or (1,inf) if $f'(y)$ at $y=1$ is greater than or less than zero, respectively. Since $y=1/z$, if $f(y)=0$ in (0,1) then $f(z)$ must be zero in (1,inf). The condition for $f(y)=0$ in (0,1) is that $f'(y)$ evaluated at $y=1$ be greater than zero, so that $f'(z)$ evaluated at $z=1$ is less than zero. The opposite statements hold true if $f(y)=0$ holds for $1<y<\text{inf}$. Since $k=z-1$, the theorem follows immediately.

These results allow a relatively simple determination of the value of k. The value of $f'(z)$ at $z=1$ can be computed from equation (15), then a search made over the appropriate range using (14) as a guide to convergence. Convergence can be accelerated by using (15) and exploiting the convexity property of $f(z)$ as well.

The value of the term $\sum_{i=1}^{n} v_i - (n-1)$ appearing in equation (16) can be interpreted as an expression of multivariate attitude towards risk. Suppose, for example, that $\sum_{i=1}^{n} v_i - (n-1) > 0$. This can be rewritten in the form

$$(1/n)\sum_{i=1}^{n} v_i > (n-1)/n \ , \tag{20}$$

implying that the decision maker would prefer a gamble with a $1/n$ chance at each $v_i = u(\underline{x}^i)$, $i=1,2\ldots,n$ to a gamble with a $1/n$ chance at $\underline{x}_*$ and a $(n-1)/n$ chance at $\underline{x}^*$, where $u(\underline{x}_*)=0$ and $u(\underline{x}^*)=1$. The decision maker then

prefers getting one of the $\underline{x}^i$ for sure rather than taking a risk of 1/n of having the "catastrophic" outcome $\underline{x}_*$ occur; in the unidimensional case, this behavior is characterized as risk averse.

It would be misleading, however, to characterize the decision maker as either risk averse or risk seeking on the basis of his attitudes toward the gamble described in (20). An individual may be risk averse in the usual sense along each of the dimensions of his utility vector when considered singly, yet still show multiattribute "risk seeking" behavior in that he may prefer the right side of equation (20) to the left. In medical problems, two dimensions may both represent serious health consequences, and the decision maker may well prefer a chance of having both together and a complementary chance of having neither over a certainty of having one of them. For example, it is perfectly reasonable to prefer the gamble A

(1 year in bed, 5 years from life) with prob.5 or
(no years in bed, no years from life) with prob. .5

to the gamble B

(1 year in bed, no years from life) with prob. .5, or
(no years in bed, 5 years from life) with prob. .5

although the decision maker may be risk averse along both the "years in bed" and the "years deducted from life" dimensions.

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